DİF. GEO I CEVAP ANAHTARI (2022-23 Final)

1) $D_X(Y+Z) = \left(X[y_1+z_1], \dots, X[y_n+z_n]\right)$

$= X[y_1]+X[z_1], \dots, X[y_n]+X[z_n]$

$= \left(X[y_1], \dots, X[y_n]\right) + \left(X[z_1], \dots, X[z_n]\right)$

$= D_X Y + D_X Z$

2) $[X,Y](f) = X(Yf) - Y(Xf)$ old. dan $f \in C(E^n, \mathbb{R})$ için

$$\left[\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_j}\right](\cancel{f}) = \frac{\partial}{\partial x_i}\left(\frac{\partial}{\partial x_j}(f)\right) - \frac{\partial}{\partial x_j}\left(\frac{\partial}{\partial x_i}(f)\right)$$

$$= \frac{\partial}{\partial x_i}\left(\frac{\partial f}{\partial x_j}\right) - \frac{\partial}{\partial x_j}\left(\frac{\partial f}{\partial x_i}\right)$$

$$= \frac{\partial^2 f}{\partial x_i \partial x_j} - \frac{\partial^2 f}{\partial x_j \partial x_i}$$

$$= 0(\cancel{f})$$

$\Rightarrow \left[\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_j}\right] = 0$ bulunur.

3) Ders notlarında Tanım ve Teorem.

4)

$F_*|_p$ dönüşümünün matrisi

$$J(F,P) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix}_P$$

$$= \begin{bmatrix} \cos x_2(P) & -x_1 \sin x_2(P) & 0 \\ \sin x_2(P) & x_1 \cos x_2(P) & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$P = \left(\frac{\pi}{2}, \frac{\pi}{2}, \pi\right)$ ise

$$J(F,P) = \begin{bmatrix} 0 & -\frac{\pi}{2} & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \ \det(J(F,P)) = -\pi/2 \neq 0$$

F regülerdir

5) $\operatorname{rot}(\operatorname{grad} f) = \nabla \wedge \operatorname{grad} f$

$$= \det \begin{bmatrix} e_1 & e_2 & e_3 \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \\ \frac{\partial f}{\partial x_1} & \frac{\partial f}{\partial x_2} & \frac{\partial f}{\partial x_3} \end{bmatrix} \quad e_i = \frac{\partial}{\partial x_i}, \ 1 \leq i \leq 3$$

$$= \underbrace{\left(\frac{\partial^2 f}{\partial x_2 \partial x_3} - \frac{\partial^2 f}{\partial x_3 \partial x_2} \right)}_{=0} e_1 - \overbrace{\left(\frac{\partial^2 f}{\partial x_1 \partial x_3} - \frac{\partial^2 f}{\partial x_3 \partial x_1} \right)}^{=0} e_2 + \underbrace{\left(\frac{\partial^2 f}{\partial x_1 \partial x_2} - \frac{\partial^2 f}{\partial x_2 \partial x_1} \right)}_{=0} e_3$$

$= 0$ dır. ($\partial x_i \partial x_j = \partial x_j \partial x_i$ olduğundan)